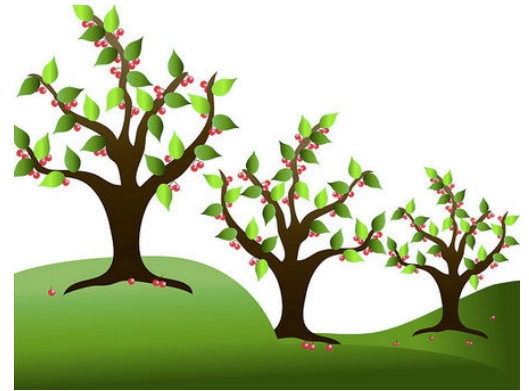


Investigation: An Orchard of Circles

Madie and Clyde have left the city in search of peace and tranquility. They recently purchased a large lot and wish to create an orchard hideout – their country escape. Their first planting will be on a circular lot with a radius of 10 units. They planted their first row of trees along an East-West line through the center of the circle placing a tree at each lattice point. (That is a tree was placed at a point where each coordinate was an integer.) They left out the tree that would have been at the circle's exact center. They continued making East-West rows as they moved North and South on their plot of land. When they were finished they noticed trees were located on the following boundary points: $(10, 0)$, $(-10, 0)$, $(0, 10)$, $(0, -10)$.



As you work on problems in this investigation, look for answers to this question:

*How can you represent circles on a coordinate plane?
How can you determine the relationship of a point and a circle?*

1. Using the grid provided, sketch a picture of Madie and Clyde's orchard and indicate where trees are planted. Compare your orchard with another group and resolve any differences.

Suppose you stand in the center of this orchard. If you look due east or north or west or south, a tree will block your line of sight. If you look in any other direction, you will be able to see out of the orchard through a gap between trees. As the trees grow, the gaps between them will narrow creating Madie and Clyde's hideout.

2. Now imagine that you remain at the center of the orchard, waiting patiently as the trees grow. In what direction should you look to be able to see out of the orchard for as long as possible? That is, what line of sight will be the last to get blocked by the growing trees? If there is more than one answer, be as general as possible.
3. Using your diagram, decide if the following points are inside the boundary, outside the boundary or exactly on the boundary. Make sure to explain how you used your diagram.
 - a. $(11, 0)$
 - b. $(9, 3)$
 - c. $(9, 2)$
 - d. $(-8, 5)$
 - e. $(8, 1)$
 - f. $(7, -6)$
 - g. $(7, -2)$
 - h. $(-9, -1)$
 - i. $(5, 4)$
 - j. $(-1, -2)$
 - k. $(-4, -6)$
 - l. $(3, -8)$
 - m. $(5, -6)$
 - n. $(0, 12)$
4. Madie and Clyde buy another circular plot of land on which to plant another orchard. They have set up coordinates as before, with center of the orchard at $(0, 0)$. They will plant trees at all points with integer coordinates that lie within the orchard, except $(0, 0)$. In this orchard, the tree at $(5, 12)$ is on the boundary. What are the coordinates of the other trees that must also be on the boundary? Explain your answer.
5. Generalize your answer for question 4. Suppose a circular orchard centered at $(0, 0)$ of any size is set up as usual with trees on lattice points.

- a. If the point (a, b) is on the lot's boundary, what other trees must also be on the boundary? Explain.
- b. If trees could be planted anywhere in this circular region defined by (a, b) , how can you describe the region with a mathematical model?



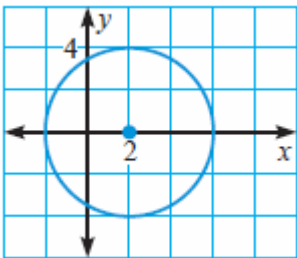
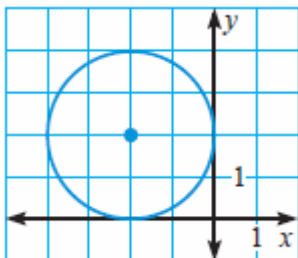
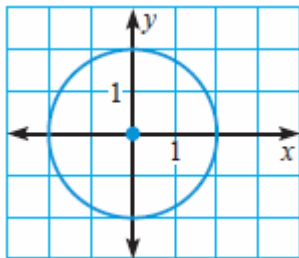
6. In another well established orchard, Clyde and Madie had a watering problem. Keeping their organizational strategies, trees are only located at lattice points. To aid their watering problem, Clyde and Maggie invested in a sprinkler system that shoots streams of water in a circular pattern. Their first sprinkler had a stream of water 4m long. Because of restriction to water and the need of a stream, the sprinkler had to be placed at (5,7). Determine if the following trees received water. Explain.
- (1, 6)
 - (2, 5)
 - (8, 2)
 - (6, 8)
 - (5, 10)
 - (7, 6)
 - (7, 7)
7. Generalize your answer from #6. Suppose the sprinkler is centered at (5, 7) and it shoots a stream of water 4m long. If the point (a, b) is on the sprinkler's boundary, what mathematical model describes the region the sprinkler covers?
8. Find a mathematical model for the sprinkler set ups below:
- Centered at (-2, 4) with a stream of water 7m.
 - Centered at (-5, -4) with a stream of water 3m.
 - Centered at (10, -2) with a stream of water 8m.
9. Clyde and Madie noticed two trees in particular that needed watering. Those two trees are at (-7, 3) and (4, 6). Upon further inspection they see that the trees between these two are rather dry. Write a mathematical model of where to place the sprinkler so that these two trees are on the boundary of the sprinkler.
10. Completely describe each of the follow "orchard" settings:
- $(x - 5)^2 + (y - 7)^2 = 9$
 - $(x + 5)^2 + (y + 7)^2 = 9$
 - $(x - 1)^2 + (y - 3)^2 = 16$



Summarize the Mathematics

- What is the equation of a circle with center at the origin and radius r ?
- What information can be determined about a circle given its equation $(x-h)^2 + (y-k)^2 = r^2$?
- Explain how to determine if a given point (a,b) is inside, outside or on a circle.

Check Your Understanding

- A bank of lights is arranged over a stage. Each light illuminates a circular area on the stage. A coordinate plane is used to arrange the lights, using the corner of the stage as the origin. The equation $(x - 13)^2 + (y - 4)^2 = 16$ represents one of the disks of light.
 - Graph the disk of light.
 - Three actors are located as follows: Henry is at (11, 4), Jolene is at (8, 5), and Martin is at (15, 5). Which actors are in the disk of light?
- Write the equation for each of the following circles:
 - 
 - 
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